

# Properties of Real Numbers Guide Notes

## PROPERTIES OF REAL NUMBERS

Let  $a$ ,  $b$ , and  $c$  be any real numbers

### 1. IDENTITY PROPERTIES

#### A. Additive Identity

The sum of any number and **0** is equal to the number. Thus, **0** is called the **additive identity**.

*For any number  $a$ , the sum of  $a$  and **0** is  $a$ .*

$$a + 0 = 0 + a = a$$

#### B. Multiplicative Identity

The product of any number and **1** is equal to the number. Thus, **1** is called the **multiplicative identity**.

*For any number  $a$ , the product of  $a$  and **1** is  $a$ .*

$$a \cdot 1 = 1 \cdot a = a$$

### 2. INVERSE PROPERTIES

#### A. Additive Inverse

The sum of any number and its opposite number (its negation) is equal to **0**. Thus, **0** is called the **additive inverse**.

*For any number  $a$ , the sum of  $a$  and  $-a$  is **0**.*

$$a + (-a) = (-a) + a = 0$$

#### B. Multiplicative Property of Zero

*For any number  $a$ , the product of  $a$  and **0** is **0**.*

$$a \cdot 0 = 0 \cdot a = 0$$

#### C. Multiplicative Inverse

The product of any number and its reciprocal is equal to **1**. Thus, the number's reciprocal is called the **multiplicative inverse**.

*For any number  $a$ , the product of  $a$  and its reciprocal  $\frac{1}{a}$  is **1**.*

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

*For any numbers  $\frac{a}{b}$ , where  $b \neq 0$ , the product of  $\frac{a}{b}$  and its reciprocal  $\frac{b}{a}$  is **1**.*

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1$$

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**Sample Problem 1:** Name the property in each equation. Then find the value of  $x$ .

- |                              |                                       |                   |
|------------------------------|---------------------------------------|-------------------|
| a. $24 \cdot x = 24$         | <b>Multiplicative identity</b>        | $x = 1$           |
| b. $x + 0 = 51$              | <b>Additive identity</b>              | $x = 51$          |
| c. $x \cdot 6 = 1$           | <b>Multiplicative inverse</b>         | $x = \frac{1}{6}$ |
| d. $x + 19 = 0$              | <b>Additive inverse</b>               | $x = -19$         |
| e. $x \cdot 7 = 0$           | <b>Multiplicative product of zero</b> | $x = 0$           |
| f. $\frac{3}{5} \cdot x = 1$ | <b>Multiplicative inverse</b>         | $x = \frac{5}{3}$ |

## 3. EQUALITY PROPERTIES

### A. Reflexive

Any quantity is equal to itself.

*For any number  $a$ ,  $a = a$ .*

$$a = a$$

### B. Symmetric

If one quantity equals a second quantity, then the second quantity equals the first quantity.

*For any numbers  $a$  and  $b$ , if  $a = b$  then  $b = a$ .*

$$a = b$$

$$b = a$$

### C. Transitive

If one quantity equals a second quantity and the second quantity equals a third quantity, then the first quantity equals the third quantity.

*For any numbers  $a$ ,  $b$ , and  $c$ , if  $a = b$  and  $b = c$ , then  $a = c$ .*

$$a = b$$

$$b = c$$

$$a = c$$

### D. Substitution

A quantity may be substituted for its equal in any expression.

*If  $a = b$ , then  $a$  may be replaced by  $b$  in any expression.*

$$a = b$$

$$3a = 3 \cdot b$$

# Properties of Real Numbers Guide Notes

**Sample Problem 2:** Evaluate  $x(xy - 5) + y \cdot \frac{1}{y}$ , if  $x = 2$  and  $y = 3$ . Name the property of equality used in each step.

|                                   |     |  |   |
|-----------------------------------|-----|--|---|
| $x(xy - 5) + y \cdot \frac{1}{y}$ | $=$ | $2(2 \cdot 3 - 5) + 3 \cdot \frac{1}{3}$ | Substitution: $x = 2$ and $y = 3$                 |
|                                   | $=$ | $2(2 \cdot 3 - 5) + 1$                   | Multiplicative inverse: $3 \cdot \frac{1}{3} = 1$ |
|                                   | $=$ | $2(6 - 5) + 1$                           | Substitution: $2 \cdot 3 = 6$                     |
|                                   | $=$ | $2(1) + 1$                               | Substitution: $6 - 5 = 1$                         |
|                                   | $=$ | $2 + 1$                                  | Multiplicative identity: $2(1) = 2$               |
| $x(xy - 5) + y \cdot \frac{1}{y}$ | $=$ | $3$                                      | Substitution: $2 + 1 = 3$                         |

## 4. COMMUTATIVE PROPERTIES

### A. Addition

The order in which two numbers are added does not change their sum.

*For any numbers  $a$  and  $b$ ,  $a + b$  is equal to  $b + a$ .*

$$a + b = b + a$$

### B. Multiplication

The order in which two numbers are multiplied does not change their product.

*For any numbers  $a$  and  $b$ ,  $a \cdot b$  is equal to  $b \cdot a$ .*

$$ab = ba$$

## 5. ASSOCIATIVE PROPERTIES

### A. Addition

The way three or more numbers are grouped when adding does not change their sum.

*For any numbers  $a$ ,  $b$ , and  $c$ ,  $(a + b) + c$  is equal to  $a + (b + c)$ .*

$$(a + b) + c = a + (b + c)$$

### B. Multiplication

The way three or more numbers are grouped when multiplying does not change their product.

*For any numbers  $a$ ,  $b$ , and  $c$ ,  $(a \cdot b) \cdot c$  is equal to  $a \cdot (b \cdot c)$ .*

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

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## Properties of Real Numbers Guide Notes

**Sample Problem 3:** Simplify variable expressions. Show all possible answers.

- a.  $6 + (x + 3)$        $= 9 + x$        $= x + 9$
- b.  $(1 + x) + 2$        $= 3 + x$        $= x + 3$
- c.  $5 \cdot 7x$        $= 35x$
- d.  $(x + 4) + 8$        $= x + 12$        $= 12 + x$
- e.  $(6)(3x)$        $= 18x$